Crystal Clear Lessons on the Microstructure of Space-Time and Modified Gravity

A team-seminar report on
Lobo, Olmo & Rubiera-Garcia, Phys. Rev. D **91**, 124001 (2015)

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Outline

1. From GR to Metric-Affine: Two Observations
2. The Crystal Analogy (paper [M])
3. Metric-Affine Dynamics: Action and Field Equations
4. The Auxiliary Metric and the Einstein Frame
5. Universality: GR as a Vacuum Attractor
6. Application I: Non-Singular Cosmologies (paper [O])
7. Application II: Wormholes as Point Defects (paper [M])
8. Summary and Discussion

The two papers behind this talk

[M] Lobo–Olmo–Rubiera-Garcia, *PRD* **91** 124001 (2015) — the "crystal" analogy + wormholes as defects.

[O] Olmo, *Introduction to Palatini theories...*, arXiv:1212.6393 — the machinery and the cosmology.

Observation 1: GR ties together two independent structures

Two different instruments.

- $g_{\mu\nu}$: lengths, angles, light cones — *no* notion of transport.
- $\Gamma^{\lambda}{}_{\mu\nu}$: parallel transport, geodesics, curvature — *no* notion of distance.
- On a differentiable manifold they are independent objects.

GR adds three assumptions — choices, not theorems

1. vanishing torsion: $\Gamma^{\lambda}{}_{\mu\nu} = \Gamma^{\lambda}{}_{\nu\mu}$;
2. metric compatibility: $\nabla_\lambda g_{\mu\nu} = 0$;
3. $\Rightarrow$ the connection is *forced* to be the Christoffel symbol.

Metric-affine = drop (3)

Let the field equations determine the connection. Equivalently, write

$$\Gamma^{\lambda}{}_{\mu\nu} = L^{\lambda}{}_{\mu\nu} + A^{\lambda}{}_{\mu\nu},$$

with A a genuine rank-3 tensor: one extra tensor field in the gravitational sector.

This is not exotic

A connection is as fundamental on a manifold as a metric is ("metric and connection are equally fundamental and independent" [O]).

Conventions: signature $(-,+,+,+)$; $\kappa^2 = 8\pi G$ when $f = \mathcal{R}$. $A \neq 0$ with $T = 0$ means pure non-metricity.

Observation 2: curvature, torsion, non-metricity — keep only Q

Quantity	Definition	Sourced by
curvature $R^{\alpha}{}_{\beta\mu\nu}$	$[\nabla_\mu, \nabla_\nu]V^\rho$	energy–momentum
torsion $T^{\lambda}{}_{\mu\nu}$	$\Gamma^{\lambda}{}_{\mu\nu} - \Gamma^{\lambda}{}_{\nu\mu}$	spin
non-metricity $Q_{\lambda\mu\nu}$	$\nabla^{L}_{\lambda} g_{\mu\nu}$	this talk

Where each theory lives

GR: $Q = T = 0$ (Riemannian).
Einstein–Cartan: $T \neq 0$.
This talk: $T = 0$, $Q \neq 0$.

Why non-metricity, and not torsion?

Torsion (dislocations) has a long and well-developed literature; non-metricity — the continuum limit of *point* defects — is largely untouched.

Note the (harmless but confusing) factor $1/2$ that [O] puts into $S^{\lambda}{}_{\mu\nu}$; [M] uses the un-halved torsion.

Step 1 — ideal crystal. Crystallographic coordinates are Cartesian, so the connection *vanishes* by definition:

$$\Gamma^{a}{}_{bc}(x) = 0.$$

Step 2 — deform to a perfect crystal. Infinitesimal displacements map as

$$dy^{\mu} = e^{\mu}{}_{i}\, dx^{i},$$

where the *distortion* $e^{\mu}{}_{i}$ satisfies $e^{\mu}{}_{i} e^{i}{}_{\nu} = \delta^{\mu}{}_{\nu}$ and $e^{i}{}_{\mu} e^{\mu}{}_{j} = \delta^{i}{}_{j}$.

Step 3 — transport of the triad.

$$e^{\alpha}{}_{i}\, \partial_{\beta} e^{i}{}_{\gamma} = \Gamma^{\alpha}{}_{\beta\gamma}(y).$$

Step 4 — metric and compatibility

Define

$$g_{\mu\nu} \equiv \delta_{ij} e^{i}{}_{\mu} e^{j}{}_{\nu}, \qquad ds^2 = g_{\mu\nu} dy^{\mu} dy^{\nu}.$$

Then the connection is *metric compatible*:

$$\nabla^{\Gamma_\lambda} g_{\mu\nu} = 0.$$

Conclusion

A defect-free crystal *is* a Riemannian manifold: "step counting" and parallel transport agree.

Step 1 — a length that ignores defects. Near a vacancy or an interstitial, "step counting" breaks down (the number of steps from A to B depends on the path), so introduce an auxiliary triad ${c^i}_\mu \neq {e^i}_\mu$ and a "pseudo-length"

$$d\tilde{s}^2 = h_{\mu\nu}\, dx^\mu dx^\nu, \qquad h_{\mu\nu} = \delta_{ij} {c^i}_\mu {c^j}_\nu.$$

Step 2 — two structures, two compatibilities. A connection ${L^\alpha}_{\beta\gamma}$ adapted to ${c^i}_\mu$ can always be found:

$$\nabla^L_\lambda h_{\mu\nu} = 0, \qquad \nabla^\Gamma{}_\lambda h_{\mu\nu} \neq 0, \qquad \nabla^L{}_\lambda g_{\mu\nu} \neq 0.$$

Step 3 — deformation. The two triads are related by the defect field

$${e^i}_\mu = {d^i}_j\, {c^j}_\mu, \qquad {d^i}_j = {\delta^i}_j \ \text{(no defects)},$$

fixed by the *kind* and *density* of defects — and defects *move*.

Golden rule

A non-zero density of point-like topological defects in the space-time microstructure ⇒ an effective metric-affine geometry. Where the defect density is low, geometry becomes Riemannian — which is why GR works so well here.

Note the two levels

- *Kinematics*: two metrics, two compatible connections (h and g, L and Γ).
- *Dynamics*: ${d^i}_j$ — and later ${\Omega_\mu}^\nu$ — are fixed by the source, and evolve with it.

The Dictionary of the Analogy

Crystal	Space-time
point defects (v. / i.)	matter fields $T_\mu{}^\nu$
defected continuum	metric-affine geometry, $\nabla^L_\lambda g_{\mu\nu} \neq 0$
auxiliary triad $c^i{}_\mu$	auxiliary metric $h_{\mu\nu}$
deformation $e^i{}_\mu = d^i{}_j c^j{}_\mu$	deformation $g_{\mu\nu} = \Omega_\mu{}^\nu h_{\alpha\nu}$
d set by defect density	$\Omega_\mu{}^\nu$ set by $T_\mu{}^\nu$
no defects: $d^i{}_j = \delta^i{}_j$	vacuum: $\Omega_\mu{}^\nu \propto \delta_\mu{}^\nu$

Two quantitative pay-offs of the analogy

- **Geometry:** the correct macroscopic language for a medium with point defects is exactly the language of Palatini gravity.
- **Estimates:** defect masses in solids are obtained from *volume comparisons* — the same construction will give us the mass of a wormhole (Sec. 7).

Terminology

Metric-affine(T=0) = Palatini: the connection is an independent dynamical variable.

Step 1: The Action and Its Variational Principle

The theory. A generic Palatini $f(\mathcal{R}, R_{\mu\nu}R^{\mu\nu})$ extension:

$$S[g, L, \psi_m] = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\, f(\mathcal{R}, R_{\mu\nu}R^{\mu\nu}) \;+\; S_m[g_{\mu\nu}, \psi_m], \tag{1}$$

with $\mathcal{R} = g^{\mu\nu}R_{\mu\nu}$, $R_{\mu\nu} = R^{\rho}{}_{\mu\rho\nu}$ and the Riemann tensor built from the independent connection L:

$$R^{\alpha}{}_{\beta\mu\nu} = \partial_\mu L^{\alpha}{}_{\nu\beta} - \partial_\nu L^{\alpha}{}_{\mu\beta} + L^{\alpha}{}_{\mu\lambda}L^{\lambda}{}_{\nu\beta} - L^{\alpha}{}_{\nu\lambda}L^{\lambda}{}_{\mu\beta}.$$

Three structural choices

- $g_{\mu\nu}$ and $L^{\lambda}{}_{\mu\nu}$ are independent fields;
- matter couples only to g (equivalence principle), $S_m = \int d^4x\sqrt{-g}\,\mathcal{L}_m$;
- $L^{\lambda}{}_{\beta\gamma} \neq L^{\lambda}{}_{\gamma\beta}$ is *allowed* while varying — i.e. torsion is admitted.

The key move

Vary w.r.t. $g^{\mu\nu}$ **and** w.r.t. $L^{\lambda}{}_{\mu\nu}$ independently $\Rightarrow$ two sets of field equations.
The theory must determine *both*, and *their relation*.

Full step-by-step derivation: Olmo 2012, arXiv:1212.6393, Sec. 1.3 (Eqs. 1.13–1.22).

Step 2: Varying the Action — the δR Identity

Varying (1) w.r.t. both fields, and writing $P_{\alpha}{}^{\beta\mu\nu} \equiv \partial f / \partial R^{\alpha}{}_{\beta\mu\nu}$:

$$\delta S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left[\left(\frac{\partial f}{\partial g^{\mu\nu}} - \frac{f}{2} g_{\mu\nu} \right) \delta g^{\mu\nu} + P_{\alpha}{}^{\beta\mu\nu} \, \delta R^{\alpha}{}_{\beta\mu\nu} \right] + \delta S_m .$$

The Palatini identity, generalised to a connection with torsion:

$$\delta R^{\alpha}{}_{\beta\mu\nu} = \nabla_{\mu} \big(\delta \Gamma^{\alpha}{}_{\nu\beta} \big) - \nabla_{\nu} \big(\delta \Gamma^{\alpha}{}_{\mu\beta} \big) + 2 S^{\lambda}{}_{\mu\nu} \, \delta \Gamma^{\alpha}{}_{\lambda\beta}, \qquad S^{\lambda}{}_{\mu\nu} \equiv \tfrac{1}{2} \big(\Gamma^{\lambda}{}_{\mu\nu} - \Gamma^{\lambda}{}_{\nu\mu} \big). \tag{2}$$

Why the extra term?

For a *symmetric* connection δR is the difference of two total derivatives. With torsion, the "difference of derivatives" no longer cancels and the torsion tensor S appears explicitly. This is precisely what will make the connection equation richer than in GR.

Dictionary reminder

$P_{\alpha}{}^{\beta\mu\nu}$ is to the connection what $T_{\mu\nu}$ is to the metric: the "source" that appears in the equation of motion of the field it multiplies.

Step 3: Integration by Parts (Surface Terms Dropped)

Only the $P\,\delta R$ piece needs work. Define $J^\mu \equiv P_\alpha{}^{\beta\mu\nu}\delta\Gamma^\alpha{}_{\nu\beta}$. Using $\nabla_\mu(\sqrt{-g}J^\mu) = \partial_\mu(\sqrt{-g}J^\mu) + 2S^\sigma{}_{\sigma\mu}\sqrt{-g}J^\mu$,

$$
\begin{aligned}
I_\Gamma &= \int d^4x\sqrt{-g}\, P_\alpha{}^{\beta\mu\nu}\nabla_\mu\big(\delta\Gamma^\alpha{}_{\nu\beta}\big) \\
&= \int d^4x\Big\{\partial_\mu\big(\sqrt{-g}J^\mu\big) - \delta\Gamma^\alpha{}_{\nu\beta}\Big[\nabla_\mu\big(\sqrt{-g}P_\alpha{}^{\beta\mu\nu}\big) - 2S^\sigma{}_{\sigma\mu}\sqrt{-g}P_\alpha{}^{\beta\mu\nu}\Big]\Big\}.
\end{aligned}
$$

Two standard manipulations

- the total derivative is a surface term $\to$ dropped (we keep $\delta\Gamma$ fixed on the boundary);
- the antisymmetric part of $P_\alpha{}^{\beta\mu\nu}$ is selected, $P_\alpha{}^{\beta[\mu\nu]} = \frac{1}{2}(P_\alpha{}^{\beta\mu\nu} - P_\alpha{}^{\beta\nu\mu})$.

The point to remember

Everything is *algebraic plus one derivative* — no $\partial\partial$ of the connection ever appears. That is *why* Palatini theories never generate higher-derivative (ghost) equations for the metric.

Step 4: The Two Field Equations

(a) Metric equation (varying $g^{\mu\nu}$):

$$\kappa^2 T_{\mu\nu} = \frac{\partial f}{\partial g^{(\mu\nu)}} - \frac{f}{2} g_{\mu\nu}, \qquad T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta S_m}{\delta g^{\mu\nu}}. \tag{3}$$

(b) Connection equation (varying $\Gamma^{\alpha}_{\ \nu\beta}$):

$$\kappa^2 H_{\alpha}^{\ \nu\beta} = -\frac{1}{\sqrt{-g}} \nabla^C_{\mu} \left(\sqrt{-g} P_{\alpha}^{\ \beta[\mu\nu]} \right) + S^{\lambda}_{\ \mu\alpha} P_{\lambda}^{\ \beta[\mu\nu]} - S^{\beta}_{\ \mu\lambda} P_{\alpha}^{\ \lambda[\mu\nu]}, \tag{4}$$

with $H_{\alpha}^{\ \nu\beta} = -\frac{1}{\sqrt{-g}} \frac{\delta S_m}{\delta \Gamma^{\alpha}_{\ \nu\beta}}$.

Simplifying assumptions (as in [M],[O])

- $H_{\alpha}^{\ \nu\beta} = 0$: matter does not couple to the connection.
- torsion set to zero: $S^{\lambda}_{\ \mu\nu} = 0$.

Torsion in one line

Energy–momentum sources curvature; spin sources torsion. It was discovered only after GR was formulated, which is the historical reason it is usually dropped — not a physical reason.

Equations (3)–(4) are Eqs. (1.20) and (1.22) of [O]; (1.21) is the form before splitting into symmetric+torsion parts.

For $f=f(\mathcal{R},\mathcal{Q})$ with $\mathcal{Q}\equiv R_{\mu\nu}R^{\mu\nu}$ and $S^{\lambda}_{\ \mu\nu}=0$ one has

$$P_{\alpha}^{\ \beta\mu\nu}=\delta_{\alpha}^{\ \mu}M^{\beta\nu},\qquad M^{\beta\nu}=f_R g^{\beta\nu}+2f_Q R^{\beta\nu}, \tag{5}$$

so that Eq. (4) collapses to the very compact statement

$$\nabla^{\Gamma}_{\alpha}\Big[\sqrt{-g}\big(f_R g^{\beta\nu}+2f_Q R^{\beta\nu}(\Gamma)\big)\Big]=0. \tag{6}$$

Notice what happened

The connection is determined by matter only: $R^{\beta\nu}$ appears inside an equation whose "coefficients" are the Lagrangian derivatives f_R, f_Q — which are functions of the matter via the field equations.

Why Second Order Matters: Metric vs Palatini

Same Lagrangian, two formalisms:

- **Metric formalism** ($\Gamma = L$ assumed): $f(\mathcal{R}, R_{\mu\nu}R^{\mu\nu})$ gives fourth-order equations; the extra derivatives propagate a ghost (Ostrogradsky instability).
- **Palatini:** $P_{\alpha}{}^{\beta\mu\nu}$ is algebraic in R, so the metric equation stays second order and no ghost appears.

Why second order ⇒ no ghosts

A Hamiltonian with at most two time derivatives per field has bounded energy; higher derivatives mean a linear-in-momentum Hamiltonian, i.e. an unbounded ("ghost") mode.

This is not an accident

Lovelock theories are celebrated precisely for giving at most second-order equations. Palatini $f(\mathcal{R}, \mathcal{Q})$ (with or without torsion) shares this property, which places it on the same footing as a "natural extension" of the Einstein–Hilbert action.

Caveat. Lovelock actions formulated à la Palatini give exactly the same equations as in the metric formulation [Borunda et al. 2008]; the difference only shows up for Lagrangians with non-linear curvature terms such as $\mathcal{R}^2, \mathcal{Q}$.

Algebraisation (i): a Matrix Equation for $\hat{P}$

Start from the metric equation (3) with the concrete $P_{\alpha}{}^{\beta\mu\nu}$ of (5):

$$f_R\, R_{\mu\nu} - \frac{f}{2} g_{\mu\nu} + 2 f_Q\, R_{\mu\alpha} R^{\alpha}_{\ \nu} = \kappa^2 T_{\mu\nu}.$$

Step 1 — raise one index with $g^{\nu\sigma}$ and define $P_{\mu}{}^{\nu} \equiv R_{\mu\alpha} g^{\alpha\nu}$. Notice the identity

$$R_{\mu\alpha} g^{\alpha\beta} R_{\beta\gamma} g^{\gamma\sigma} = P_{\mu}{}^{\alpha} P_{\alpha}{}^{\sigma} = (P^2)_{\mu}{}^{\sigma},$$

so that the metric equation becomes a purely matrix equation

$$f_R\, P_{\mu}{}^{\nu} - \frac{f}{2} \delta_{\mu}{}^{\nu} + 2 f_Q\, (P^2)_{\mu}{}^{\nu} = \kappa^2 T_{\mu}{}^{\nu} \qquad \Longleftrightarrow \qquad \boxed{2 f_Q \hat{B}^2 + f_R \hat{B} - \frac{f}{2} \hat{I} = \kappa^2 \hat{T}}, \qquad \hat{B} \equiv \hat{P}.$$

Why this is a big deal

$R_{\mu\nu}$ has been traded for the matrix $P_{\mu}{}^{\nu} = R_{\mu\alpha} g^{\alpha\nu}$ multiplying itself. There are no derivatives left in this equation — it is algebra, not calculus.

Notation warning. The same object is called $P_{\mu}{}^{\nu}$ in [M] and $B_{\mu}{}^{\nu}$ in [O]; do not let the letters confuse you.

Step 1 is Eq. (1.42) of [O].

Algebraisation (ii): Completing the Square

Starting from $2f_Q\hat{B}^2 + f_R\hat{B} - \frac{f}{2}\hat{I} = \kappa^2\hat{T}$, complete the square in $\hat{P}$ and multiply by $2f_Q$:

$$2f_Q\left(\hat{P} + \frac{f_R}{4f_Q}\hat{I}\right)^2 = \left(\frac{f}{2} + \frac{f_R^2}{8f_Q}\right)\hat{I} + \kappa^2\hat{T}. \tag{7}$$

The single most important equation of the talk

Since $\mathcal{R} = \mathrm{Tr}\,[\hat{P}]$ and $\mathcal{Q} = \mathrm{Tr}\,[\hat{P}^2]$, the *left* side is algebraic in $\hat{P}$, while the *right* side contains only the matter stress tensor and the two scalars $\mathcal{R}, \mathcal{Q}$. Therefore

$$\boxed{\hat{P} = \hat{P}(\hat{T})} \qquad \mathcal{R} = \mathcal{R}(\hat{T}), \quad \mathcal{Q} = \mathcal{Q}(\hat{T}).$$

Curvature is slaved to matter, algebraically.

Keep the two kinds of "algebra" apart

- $\hat{P}$ is a *matrix*; its powers $\hat{P}^2$ are matrix powers.
- $\mathcal{R}$, $\mathcal{Q}$ are the *traces* $\mathrm{Tr}\,[\hat{P}]$, $\mathrm{Tr}\,[\hat{P}^2]$ — ordinary numbers.

Eq. (4) of [M]; Eqs. (1.42)–(1.43) of [O].

Perfect Fluids (i): Work in the Fluid Rest Frame

For $T_{\mu\nu} = (\rho + P)u_\mu u_\nu + P g_{\mu\nu}$ (a sum of perfect fluids), Eq. (7) becomes

$$2f_Q \left(\hat{P} + \frac{f_R}{4f_Q}\hat{I}\right)^2 = \left(\kappa^2 P + \frac{f}{2} + \frac{f_R^2}{8f_Q}\right)\hat{I} + \kappa^2(\rho + P)u_\mu u^\mu \tag{8}$$

with $T_\mu{}^\nu = \mathrm{diag}(-\rho, P, P, P)$ in the fluid rest frame.

Defining

$$\lambda^2 \equiv \kappa^2 P + \frac{f}{2} + \frac{f_R^2}{8f_Q}, \tag{9}$$

the matrix equation becomes diagonal:

$$2f_Q \left(\hat{P} + \frac{f_R}{4f_Q}\hat{I}\right)^2 = \mathrm{diag}\big(\lambda^2 - \kappa^2(\rho + P),\ \lambda^2,\ \lambda^2,\ \lambda^2\big) \tag{10}$$

Why the time entry is singled out

Diagonal matrices can be square-rooted entry by entry — and the fluid velocity $u_\mu u^\mu$ singles out the time direction. That is the origin of the $\sigma_1 \neq \sigma_2$ asymmetry found on the next slide, and ultimately of the fact that h and g differ by a *deformation*, not a conformal factor.

Eqs. (1.41)–(1.44) of [O].

$$\hat{P} = \frac{1}{\sqrt{2f_Q}}\,\mathrm{diag}\Big(\sqrt{\lambda^2 - \kappa^2(\rho+P)},\, \lambda\hat{I}_{3\times3}\Big) - \frac{f_R}{4f_Q}\hat{I}$$

Then $\hat{\Sigma} \equiv f_R\hat{I} + 2f_Q\hat{P}$ is diagonal,

$$\hat{\Sigma} = \mathrm{diag}\big(\sigma_1, \sigma_2, \sigma_2, \sigma_2\big)$$

with

$$\sigma_1 = \frac{f_R}{2} \pm \sqrt{2f_Q}\,\sqrt{\lambda^2 - \kappa^2(\rho+P)}, \qquad \sigma_2 = \frac{f_R}{2} + \sqrt{2f_Q}\,\lambda$$

Read this carefully — two warnings

(1) No conformal factor. $\sigma_1 \neq \sigma_2$: the time and the space directions are scaled *differently*. So g and h are related by a genuine deformation.

(2) The two signs are not two theories. They are the two roots of the same algebraic equation. At high density one must switch branch to keep σ_1 continuous and differentiable — and this switch is what makes the bounce work (Sec. 6).

Eqs. (1.45)–(1.47) of [O]. The "$\pm$" is kept in the literature precisely because of warning (2).

Step 3 — the connection equation becomes a compatibility statement. With $\hat{\Sigma} = f_R \hat{I} + 2 f_Q \hat{P}$ — now a function of the *matter* — Eq. (6) reads

$$\nabla^{\Gamma}_{\alpha}\Big[\sqrt{-g}\, g^{\beta\lambda}\Sigma_{\lambda}{}^{\nu}\Big] = 0.$$

Define h by demanding that this be a metric-compatibility condition:

$$\begin{aligned} &\sqrt{-g}\, g^{\beta\lambda}\Sigma_{\lambda}{}^{\nu} = \sqrt{-h}\, h^{\beta\nu} \\ \Longrightarrow \quad &\nabla^{\Gamma}_{\alpha}\big[\sqrt{-h}\, h^{\beta\nu}\big] = 0. \end{aligned} \tag{11}$$

Then the independent connection is a Christoffel symbol — but of h, not of g:

$$\Gamma^{\alpha}{}_{\beta\gamma} = \frac{h^{\alpha\rho}}{2}\big(\partial_\beta h_{\rho\gamma} + \partial_\gamma h_{\rho\beta} - \partial_\rho h_{\beta\gamma}\big). \tag{12}$$

Crystal reading

$h_{\mu\nu} \leftrightarrow$ the *ideal lattice* you get by ignoring the defects; the connection L of the defected crystal is its compatible connection.

Vocabulary

$g_{\mu\nu}$: the physical metric — matter couples to it.
$h_{\mu\nu}$: the auxiliary metric — the one compatible with the connection.

Eqs. (1.35)–(1.37) of [O].

Equivalently, g and h differ by a deformation (not by a conformal factor):

$$g_{\mu\nu} = \Omega_\mu{}^\nu \, h_{\alpha\nu}, \qquad \Omega_\mu{}^\nu = \frac{f_R \, \delta_\mu{}^\nu + 2 f_Q P_\mu{}^\nu}{\sqrt{\det\left(f_R \hat{I} + 2 f_Q \hat{P}\right)}}. \tag{13}$$

- $\Omega_\mu{}^\nu = \Omega_\mu{}^\nu(\hat{T})$: the deformation is fixed by the *matter* distribution — exactly as the defect field $d^i{}_j$ is fixed by the defect density.
- Mind the index order: $g_{\mu\nu} = \Omega_\mu{}^\nu h_{\alpha\nu}$, i.e. Ω has one upper and one lower index.

h is *not* a second space-time

It is the variable in which the equations become simple. One may always rewrite everything in terms of g — at the price of carrying the extra tensor $A^\lambda{}_{\mu\nu} = \Gamma - L$, whose equations are far uglier.

A concrete by-product

In a Bianchi I universe this deformation will have *different* factors in the time and space directions (σ_1 vs. σ_2) — a purely matter-induced anisotropy of the effective geometry.

Eqs. (1.37)–(1.39) of [O]; Eq. (5) of [M].

Inserting (11)–(13) back into the metric equation gives equations whose left-hand side is the Einstein tensor *of the auxiliary metric*:

Form of [M]

$$G_\mu{}^\nu(h) = \kappa^2 \left|\hat{\Omega}\right|^{1/2} \left[T_\mu{}^\nu - \tfrac{1}{2}\delta_\mu{}^\nu \left(T + 2L_G\right)\right], \quad (14)$$

with $L_G \equiv f/(2\kappa^2)$ and $T \equiv g^{\mu\nu}T_{\mu\nu}$.

Equivalent form of [O]

$$R_\mu{}^\nu(h) = \frac{1}{\sqrt{\det \hat{\Sigma}}} \left[\frac{f}{2}\delta_\mu{}^\nu + \kappa^2 T_\mu{}^\nu\right]. \quad (15)$$

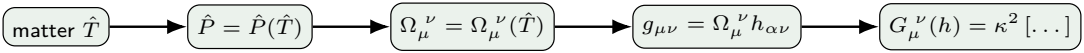
What we gained

- h obeys second-order Einstein-like equations $\Rightarrow$ standard solution techniques apply;
- the geometry is matter-sensitive: it responds to the *type* and *density* of the sources;
- in vacuum the source terms collapse and we are back to GR.

Consistency check (a one-line exercise)

Since $\hat{\Omega} = \hat{\Sigma}/\sqrt{\det \hat{\Sigma}}$ one has $|\hat{\Omega}|^{1/2} = 1/\sqrt{\det \hat{\Sigma}}$: taking the trace of (15) and subtracting $\frac{1}{2}\delta R$ reproduces (14). The two papers are writing the same equation.

The Chain of Logic (and the $f(\mathcal{R})$ Limit)



Limit $f_Q \to 0$: Palatini $f(\mathcal{R})$

$\sigma_1 \to \sigma_2 \to f_R, \qquad S \to \Omega \to f_R, \qquad h_{\mu\nu} = f_R\, g_{\mu\nu}$

i.e. h is now a conformal rescaling of g, and (15) reduces to the known Palatini $f(\mathcal{R})$ equation

$$f_R\, R_{\mu\nu}(h) - \frac{f}{2} g_{\mu\nu} = \kappa^2 T_{\mu\nu}.$$

Interpretation

All the new physics sits in the two "dressing functions" $\sigma_1(\rho, P)$ and $\sigma_2(\rho, P)$: they encode how the effective geometry responds to matter.

At low density $\sigma_{1,2} \to f_R \to 1$ and we recover GR; deviations are controlled by $\kappa^2\rho/R_P$ — i.e. they are Planck-suppressed.

Set $\hat{T} = 0$ in Eq. (7). The right-hand side becomes proportional to the identity, so the matrix equation forces

$$\hat{P} = C(\mathcal{R}, \mathcal{Q})\, \hat{I}, \qquad C(\mathcal{R}, \mathcal{Q}) \equiv \sqrt{\left(\frac{f}{2} + \frac{f_R^2}{8 f_Q}\right) \Big/ 2 f_Q} \; - \; \frac{f_R}{4 f_Q} \tag{16}$$

Trace logic. Taking the trace and the trace of the square of $\hat{P} = C\hat{I}$ in four dimensions,

$$\mathcal{R} = \mathrm{Tr}\,[\hat{P}] = 4C(\mathcal{R}, \mathcal{Q}), \quad \mathcal{Q} = \mathrm{Tr}\,[\hat{P}^2] = 4C^2(\mathcal{R}, \mathcal{Q})$$

These are two equations for two unknowns: only particular constant values $\mathcal{R}_v, \mathcal{Q}_v$ satisfy them — the *vacuum configurations*; $C(\mathcal{R}_v, \mathcal{Q}_v)$ is then a constant.

Then the deformation collapses

$\hat{P} = C\hat{I}$ in (13) gives

$$\Omega_\mu{}^\nu = \frac{1}{(f_R + 2 f_Q C)\big|_v}\, \delta_\mu{}^\nu \qquad \Longrightarrow \qquad R_{\mu\nu}(h) = R_{\mu\nu}(g) \tag{17}$$

i.e. g and h differ only by a constant factor — absorbable into a global redefinition of units.

Inserting (17) into (14) makes the matter terms vanish and leaves a pure cosmological-constant equation:

$$G_{\mu\nu}(g) = -\frac{\kappa^2 L_G}{\big(f_R + 2f_Q C\big)\big|_v}\, g_{\mu\nu} \equiv -\Lambda_{\rm eff}\, g_{\mu\nu}, \qquad \Lambda_{\rm eff} = \frac{\kappa^2 L_G}{\big(f_R + 2f_Q C\big)\big|_v}. \tag{18}$$

Two logically independent facts

1. **Metricity is restored**: $\nabla^{\Gamma_\lambda} g_{\mu\nu} = 0$ holds in vacuum, because the deformation is (proportional to) the identity.
2. **The dynamics is that of GR**: Einstein's equations with an effective cosmological constant.

Generic, not a coincidence

The same vacuum limit GR $+\Lambda_{\rm eff}$ is found for

- Palatini $f(\mathcal{R}, \mathcal{Q})$ (any f);
- Palatini Born–Infeld, $f = |\hat{\Omega}|^{1/2}$;
- BI + an $f(\mathcal{R})$ term;
- $f(|\hat{\Omega}|)$ deformations of BI;
- massive-gravity-inspired Palatini families.

[O], Sec. 1.5; [M], Sec. IV.

Reading the Attractor Physically

- Where the defect (matter) density is low, the geometry becomes Riemannian: $\Omega_{\mu}^{\ \nu} \to \delta_{\mu}^{\ \nu}$, up to an irrelevant constant rescaling.
- $\nabla^{\Gamma_\lambda} g_{\mu\nu} = 0$ is *recovered* in vacuum — metricity is an emergent property, not an assumed one.
- Λ_{eff} need not be strictly constant: it arises as the *vacuum limit of a function*. This reframes the traditional cosmological-constant problem.
- Einstein's equations look like a universal low-density attractor, not a unique starting axiom.

Sharp contrast with the standard lore

"Extensions of GR necessarily introduce new dynamical degrees of freedom" — **not true** in the metric-affine framework. New high-energy physics can arise *non-perturbatively* without new propagating modes.

But be honest about what is *not* understood

Why the vacuum GR $+\Lambda_{\text{eff}}$ attractor appears is not fully understood; it is observed across many families of theories. This is an open question, in the authors' own words.

Related review: Olmo & Rubiera-Garcia, arXiv:2203.04116 (Ricci-based gravity: the modern unified formulation of the same structure).

Why Bother With Early Cosmology?

- GR predicts its own breakdown: the big bang singularity, where classical curvature invariants diverge.
- A full quantum theory of gravity is not available, and its signatures may well be washed out by inflation.
- **A conservative middle road:** find an effective classical, non-singular geometry on which quantum fields can still be treated perturbatively.

Bounce scenarios

Replace the singular bang by a bounce: a contracting phase turns into the expanding phase we live in. Away from the bounce the two branches must asymptote an effective GR geometry.

Why metric-affine is interesting here

- *No* new dynamical degrees of freedom, *no* higher derivatives.
- New effects come from the non-trivial role matter plays in determining the connection $\rightarrow$ non-linear matter dynamics at very high energy density.
- Vacuum = GR exactly: a genuinely *minimal* extension.

[O], Sec. 1.5; the extended analysis of Barragan & Olmo (2010) to several perfect fluids is what this chapter adds.

Setup. Three different scale factors (to test robustness against anisotropy):

$$ds^2 = -dt^2 + \sum_{i=1}^{3} a_i^2(t)\,(dx^i)^2. \qquad (19)$$

Solving (13) with $\hat{\Sigma} = \mathrm{diag}(\sigma_1, \sigma_2, \sigma_2, \sigma_2)$ gives the auxiliary metric

$$h_{tt} = -\frac{\sigma_2^2}{\sqrt{\sigma_1\sigma_2}} \equiv -S, \quad h_{ij} = \underbrace{\sqrt{\sigma_1\sigma_2}}_{\equiv\,\Omega}\, a_i^2\delta_{ij}. \qquad (20)$$

Time and space are scaled differently when $\sigma_1 \neq \sigma_2$.

The expansion and shear equations collapse into

$$\mathcal{H}^2 = \frac{1}{6\sigma_1}\frac{\left[f + \kappa^2(\rho + 3P) - \dfrac{6K\sigma_2}{a^2}\right]}{\left[1 + \frac{3}{2}\Delta_1\right]^2}, \qquad \Delta_1 \equiv -\sum_i (1 + w_i)\rho_i \frac{\partial_{\rho_i}\Omega}{\Omega}. \qquad (21)$$

Compare with Friedmann

Standard GR: $\mathcal{H}^2 = \frac{\kappa^2}{3}\rho$. Here everything that differs from GR is a function of the matter: f, σ_1, Δ_1. Low density: $\sigma_1 \to 1$, $\Delta_1 \to 0$, $\Omega \to 1$, and Friedmann is recovered.

Eqs. (1.52)–(1.72) of [O].

Palatini $f(\mathcal{R})$: a Bounce Without Exotic Matter

The model. $f(\mathcal{R}) = \mathcal{R} + a\mathcal{R}^2/R_P$, with $R_P = 1/\ell_P^2$ (Planck curvature). Its trace equation $Rf_R - 2f = \kappa^2 T$ gives $R = -\kappa^2 T$, *the same relation as GR*, so the theory deviates from GR only when $\rho \sim \rho_P \equiv R_P/\kappa^2$.

Bounce condition. A bounce means $\theta = \dot{V}/(3V) = 0$. For a single fluid of constant w this happens when

$$\rho_B = \frac{\rho_P}{2a(3w-1)} > 0, \tag{22}$$

equivalently $\boxed{f_R = 1 + \dfrac{2aR}{R_P} = 0}$

- $a > 0$: fluids with $w > 1/3$ avoid the singularity.
- $a < 0$: fluids with $w < 1/3$ — including pressureless dust, $w = 0$ — avoid singularity.

The physically important point

A bounce is obtained without any exotic matter that violates the energy conditions. The reason: at high energy density, gravity can become *repulsive* for sources with $w > -1$, while remaining attractive at low densities for the same sources.

(Pure radiation, $w = 1/3$, is a special case: it produces no modification at all in Palatini $f(\mathcal{R})$ — but the trace anomaly of the electromagnetic field makes $w_{\text{eff}}^{rad} \neq 1/3$ in a real photon gas.)

The Price: the Shear Diverges

For $f(\mathcal{R})$, the shear of a congruence of comoving observers is

$$\sigma^2 = \frac{\left(C_{12}^2 + C_{23}^2 + C_{31}^2\right)}{9 f_R^2} \left(\frac{V_0}{V(t)}\right)^2 \propto \frac{1}{f_R^2}, \tag{23}$$

where C_{ij} are anisotropy constants $(C_{12} + C_{23} + C_{31} = 0)$.

Since the bounce *requires* $f_R = 0$, an anisotropic universe inevitably develops a divergent shear at the bounce. This is generic for any $f(\mathcal{R})$ whose f_R vanishes somewhere, whatever the fluid content.

The cure: add $\mathcal{Q}$

With a $\mathcal{Q} = R_{\mu\nu}R^{\mu\nu}$ term there is a second bounce mechanism: $\mathcal{Q}$ is bounded from above, and the bounce can happen at $\mathcal{Q} = \mathcal{Q}_{\max}$ instead of at $f_R = 0$ — so the shear need not diverge.

$$f(\mathcal{R},\mathcal{Q}) = \mathcal{R} + a\frac{\mathcal{R}^2}{R_P} + b\frac{\mathcal{Q}}{R_P}, \qquad b > 0. \quad (24)$$

- $b > 0$ makes $\mathcal{Q}$ bounded from above for $w > -1$: there is a maximum density. (This is what a *positive* $\mathcal{Q}$ term buys us.)
- Two bounce mechanisms:
  (i) $\mathcal{Q} = \mathcal{Q}_{\max}$
  (ii)The square root vanishes

The technical point that makes it work

The two branches $\sigma_1^{\pm}$ must be glued so that σ_1 stays continuous and differentiable across the point where the square root vanishes. With only one branch, one would either lose the regular solutions or hit a discontinuity in the middle of the evolution.

Why this mechanism instead of the former

In pure $f(\mathcal{R})$ the bounce *requires* $f_R = 0$, which forces the shear to diverge. The $\mathcal{Q} = \mathcal{Q}_{\max}$ route is independent of $f_R = 0$, so the shear can stay finite.

Bouncing solutions for a fluid of constant w

Class I ($a \geq 0$): bounce for

$$w > \frac{a}{2+3a},$$

so in particular radiation always bounces.

Class II ($a \leq 0$): bounce with branch switching. For $-1/4 < a \leq 0$ the allowed range is

$$-\frac{1}{3} + \frac{1}{3}\sqrt{\frac{1+4a}{1+a}} < w < \infty,$$

which extends down to $w \to -1/3$ as $a \to -1/4$.

Punch line for anisotropic universes

For $a < 0$ the interval $0 \leq w \leq 1/3$ always lies inside the family of completely regular isotropic *and* anisotropic bouncing solutions — dust and radiation included. For $a \geq 0$ the radiation case is always non-singular too.

Radiation universe: fully analytic

For $f = \mathcal{R} - \frac{\ell_P^2}{2}\mathcal{R}^2 + \ell_P^2\mathcal{Q}$,

$$\kappa^2\rho_{\max} = \frac{3R_P}{16}, \qquad \mathcal{Q}_{\max} = \frac{3R_P^2}{16},$$

and the expansion vanishes there however large the anisotropy is. At low density one recovers the GR result $\mathcal{Q} = 3P^2 + \rho^2$.

Bonus: a Bridge to Quantum Gravity

A curious exact correspondence

The effective dynamics of loop quantum cosmology in a Friedmann–Robertson–Walker background filled with a massless scalar field can be *exactly reproduced* by a Palatini $f(\mathcal{R})$ theory (Olmo & Singh, JCAP 0901, 030 (2009)).

Extending that result to more general space-times and matter sources could shed light on the relation between (24) and a more fundamental theory of quantum gravity.

Where the model stands

- proposed on grounds of *mathematical simplicity*;
- motivated by the form of effective actions from perturbative quantisation in curved backgrounds;
- successful for both cosmological *and* black-hole singularities [O], Sec. 1.6;
- still lacks a first-principles derivation — a fair open problem.

Wheeler's geon (1955)

A singularity-free, classical, self-gravitating object — for instance a "ball of light" held together by its own gravitational field. Geons are solutions of the *sourceless* Einstein–Maxwell equations.

Misner–Wheeler (1957)

With non-trivial topology one can build particle-like objects with mass and charge without point-like sources. This is the seed of the idea that space-time topology could play a fundamental role.

The "quantum foam" picture

Combine (i) geons, (ii) spontaneous particle–antiparticle creation, and you are led to a Planck-scale space-time in which structures such as wormholes are continuously created and annihilated.

Wheeler's estimate for a metric fluctuation,

$$\Delta g \sim \frac{(\hbar G/c^3)^{1/2}}{L},$$

becomes appreciable only for tiny L.

The open question: how does such a microstructure turn into the smooth continuum we see? [M], Sec. I.

Quadratic Gravity: the Area Function

Setup. Quadratic Palatini gravity coupled to a sourceless electric field:

$$f = \mathcal{R} + \ell_P^2\left(a\mathcal{R}^2 + bR_{\mu\nu}R^{\mu\nu}\right), \qquad \mathcal{L}_m = -\tfrac{1}{2}F_{\mu\nu}F^{\mu\nu}.$$

Solving the field equations, the point-like central singularity of the Reissner–Nordström solution is replaced by an extended structure: the 2-spheres have area $A = 4\pi r^2(x)$ with

$$r^2(x) = \frac{x^2 + \sqrt{x^4 + 4r_c^4}}{2}, \qquad r_c = \sqrt{\ell_P\, r_q}, \qquad r_q^2 = 2Gq^2 \tag{25}$$

Read off the two limits:

$$x \to 0: \quad r^2 \to r_c^2 \ \ (\text{a } \textit{minimum}), \qquad |x| \text{ large}: \quad r^2 \simeq x^2 + \frac{r_c^4}{x^2}$$

Why this is remarkable

- The area *never* goes to zero: the "centre" is at finite area.
- For specific charge-to-mass ratio the geometry at $x = 0$ is completely smooth.
- Nothing forces us to stop at $x = 0 \Rightarrow$ extend to $x \in (-\infty, +\infty)$.

Consequence

$x = 0$ becomes a wormhole throat: a bridge between two $x > 0/x < 0$ halves, each asymptotic to the RN geometry.

Eqs. (9) and (11) of [M]; the explicit solution is in Olmo & Rubiera-Garcia, PRD 86, 044014 (2012).

A Geon for Arbitrary Charge and Mass

- The electric flux at $x = 0$ has universal (topological) properties: the wormhole structure exists for arbitrary values of the charge and mass parameters.
- This is so *even though* curvature scalars built from $g_{\mu\nu}$ typically diverge at $x = 0$: probe physics (flux, action, energy) stays finite.
- There is no point-like source: only electric flux trapped in non-trivial topology.
- Using a simple model of charge collapse, these wormholes can be generated *dynamically* out of Minkowski space — supporting the idea of topology change on which the foam picture rests.

Defect interpretation

A wormhole *is* a point defect of the space-time microstructure:

- it is an extended, non-singular structure;
- it carries the "charge" of the defect;
- its size will be set by a microscopic length — like a vacancy in a lattice.

The same geonic solutions appear in Palatini Born–Infeld gravity (Olmo, Rubiera-Garcia & Sanchis-Alepuz, EPJC 74, 2804 (2014)).

Born–Infeld Gravity: Volume Mismatch as Defect Density

$$S_{\rm BI} = \frac{1}{2\kappa^2\ell_\epsilon^2}\int d^4x\big(\sqrt{-g}-\sqrt{-q}\big) + S_m, \qquad q_{\mu\nu} = g_{\mu\nu} - 2\ell_\epsilon^2 R_{\mu\nu} \tag{26}$$

Step 1 — connection. The connection equation $\nabla^{\Gamma}{}_{\alpha}[\sqrt{-q}\,q^{\mu\nu}] = 0$ says that $\Gamma^{\alpha}_{\mu\nu}$ is the Levi-Civita connection of the auxiliary metric $q_{\mu\nu}$; in vacuum $q_{\mu\nu} \propto g_{\mu\nu}$.

Step 2 — volume mismatch as a defect measure:

$$\underbrace{\sqrt{-g}-\sqrt{-q}=0}_{\text{vacuum: no point defects}}, \qquad \underbrace{\sqrt{-g}-\sqrt{-q}=-\sqrt{-g}\big(1-|\hat{\Omega}|^{1/2}\big)}_{\text{matter on: defects present}}$$

This is exactly the solid-state construction

In a crystal, the mass of a point defect is measured by comparing the volume of the defected structure with the volume of the ideal one. Here:

$$\int d^4x\big(\sqrt{-g}-\sqrt{-q}\big)$$

is that comparison — the BI gravitational action is literally a volume-audit of the space-time microstructure.

The $\ell_\epsilon \to 0$ limit

The leading term of $\big(1-|\hat{\Omega}|^{1/2}\big)$ reduces to $\mathcal{R}$ itself. So: the Einstein–Hilbert action is the zeroth-order effect of microscopic defects.

The Mass of a Defect: the Key Formula

$$\int d^3x\, E_{\rm BI} = 2Mc^2\,\frac{\delta_1}{\delta_c}, \qquad \delta_1 \equiv \frac{1}{2r_S}\sqrt{\frac{r_c^3}{\ell_\epsilon}}, \qquad \delta_c \equiv \frac{3\,\Gamma[3/4]^2}{\sqrt{2}\,\pi^{3/2}} \approx 0.572 \tag{27}$$

Writing $S_{\rm BI} = \int dt\, d^3x\, E_{\rm BI}$ and evaluating on electromagnetic Maxwell solutions:

- $r_S = 2M$ is the Schwarzschild radius and δ_1 plays the role of a charge-to-mass ratio.
- $\delta_1 = \delta_c$: the regular solutions mentioned before; $\delta_1 \neq \delta_c$: curvature scalars diverge at the throat — yet the effective energy is always finite.

What the formula says

The action evaluated on the solution equals the action of a point-like particle of mass $2M\delta_1/\delta_1^*$ at rest. Exactly as in condensed matter, the mass of the defect is obtained by comparing invariant volumes — but here the "defect" is a wormhole.

Eq. (11) of [M]. The factor 2 comes from the wormhole having *two* identical sides.

Scale of the Defect

- The size of the wormhole is measured in units of ℓ_ϵ, the length scale that characterises the microscopic structure of space-time — the lattice spacing of the analogy.
- In terms of the number of charges (in proton units),

$$r_q = \frac{2\ell_P N_q}{N_c}, \qquad N_c \equiv \sqrt{2/\alpha_{em}} \approx 16.55. \tag{28}$$

- Therefore

$$r_c = \sqrt{r_q \ell_\epsilon} \;\propto\; N_q^{1/2}(\ell_P \ell_\epsilon)^{1/2}, \qquad \ell_\epsilon = \ell_P \;\Rightarrow\; r_c \propto N_q^{1/2}\ell_P$$

Reading (28) physically

A wormhole is "a hole in the space-time structure whose size is measured in the natural units of an analogous microscopic lattice". The electric field both creates the hole *and* defines a defect density through the non-metricity associated with its energy density: the object has local *and* global properties.

Order-of-magnitude exercise

For a single elementary charge, $N_q \simeq 1$ and $r_c \sim \ell_P$: the defect is Planck-sized. Any observable effect therefore requires an enormous N_q — or a modification of the microscopic scale ℓ_ϵ.

One Wormhole, Two Charges: a Hint of Entanglement

- Observers on one mouth see a field as if generated by positive charges; observers on the other mouth see negative charges.
- A *single* structure simultaneously represents two apparent opposites — exactly like interstitials and vacancies in a crystal.
- The factor of 2 in (27) is precisely because the wormhole has two identical sides.

Honesty check

This is a *suggestive* correspondence, not a derivation of ER=EPR. What it does establish is that a single topological object can carry two opposite charge labels — which is already non-trivial.

Entanglement as a geometric statement

The apparent non-locality of entanglement transfer can be re-read as: acting on one of the manifestations of the wormhole necessarily has an instantaneous effect on the other. The relation between wormhole geometry and entanglement was first suggested by Maldacena & Susskind (2013).

Where it fits in our discussion

If matter is the coarse-grained description of space-time defects, then entanglement between matter subsystems may admit a defect-geometry reading. This is one of the more speculative but also more interesting threads of the programme.

Summary: Five Lessons

1. **Geometry can be emergent.** A microscopic structure (quantum foam, defects) can give rise to an effective metric-affine geometry at large scales.
2. **Non-metricity matters.** Point-like defects in crystals $\leftrightarrow$ non-metricity in space-time; the matter stress–energy tensor plays the role of the defect density. Dislocations $\leftrightarrow$ torsion.
3. **Auxiliary metric.** The independent connection turns out to be the Levi-Civita connection of an auxiliary metric $h_{\mu\nu}$ (or $q_{\mu\nu}$), related to $g_{\mu\nu}$ by a matter-dependent deformation $\Omega_{\mu}^{\ \nu}$ — not a conformal factor.
4. **GR is an attractor.** In vacuum a vast family of metric-affine theories reduces to Einstein's equations with Λ_{eff}, *without* new propagating degrees of freedom and *without* higher derivatives.
5. **New objects, new physics.** Wormholes = point defects: non-singular, topologically protected, with a defect "mass" obtained by the same volume comparison used in solid-state physics.

Methodological moral

Lessons from condensed-matter physics can be powerful guides for modified gravity — here they convert a formal statement about connections into a physical picture with quantitative content.

References I

- **[M]** F. S. N. Lobo, G. J. Olmo, D. Rubiera-Garcia, *Crystal clear lessons on the microstructure of space-time and modified gravity*, Phys. Rev. D **91** (2015) 124001, arXiv:1412.4499 [hep-th].
- **[O]** G. J. Olmo, *Introduction to Palatini theories of gravity and nonsingular cosmologies*, chapter in *Open Questions in Cosmology* (InTech, 2012), arXiv:1212.6393 [gr-qc].
- G. J. Olmo, D. Rubiera-Garcia, *Some Recent Results on Ricci-Based Gravity Theories*, WSPC proceedings (2022), arXiv:2203.04116 [gr-qc].
- G. J. Olmo, *Palatini approach to modified gravity: $f(R)$ theories and beyond*, Int. J. Mod. Phys. D **20** (2011) 413.
- G. J. Olmo, H. Sanchis-Alepuz, S. Tripathi, *Dynamical aspects of generalized Palatini theories...*, Phys. Rev. D **80** (2009) 024013, arXiv:0907.2787 [gr-qc].
- C. Barragan, G. J. Olmo, *Isotropic and anisotropic bouncing cosmologies in Palatini $f(R)$ theories*, Phys. Rev. D **82** (2010) 084015, arXiv:1005.4136 [gr-qc].
- G. J. Olmo, D. Rubiera-Garcia, *Nonsingular black holes in quadratic Palatini gravity*, Eur. Phys. J. C **72** (2012) 2098.
- G. J. Olmo, D. Rubiera-Garcia, H. Sanchis-Alepuz, *Geonic black holes and remnants...*, Eur. Phys. J. C **74** (2014) 2804.
- F. S. N. Lobo, J. Martinez-Asencio, G. J. Olmo, D. Rubiera-Garcia, *Dynamical generation of wormholes with charged fluids...*, Phys. Rev. D **90** (2014) 024033; Phys. Lett. B **731** (2014) 163.

References II

- F. S. N. Lobo, G. J. Olmo, D. Rubiera-Garcia, *Microscopic wormholes and the geometry of entanglement*, Eur. Phys. J. C **74** (2014) 2924.
- G. J. Olmo, P. Singh, *Covariant effective action for loop quantum cosmology from order reduction*, JCAP **0901** (2009) 030, arXiv:0806.2783 [gr-qc].
- M. Bañados, P. G. Ferreira, *Eddington-inspired Born-Infeld gravity*, Phys. Rev. Lett. **105** (2010) 011101.
- J. Maldacena, L. Susskind, *Cool horizons for entangled black holes*, Fortsch. Phys. **61** (2013) 781.
- F. W. Hehl, J. D. McCrea, E. W. Mielke, Y. Ne'eman, *Metric-affine gauge theory of gravity*, Phys. Rept. **258** (1995) 1.
- M. O. Katanaev, I. V. Volovich, *Theory of defects in solids and three-dimensional gravity*, Ann. Phys. **216** (1992) 1.

Thank you!

Backup 1: the Connection Equation With Torsion

Keeping torsion, the connection equation (4) reads, in the "∇^C" form of [O],

$$\kappa^2 H_\alpha^{\ \nu\beta} = -\frac{1}{\sqrt{-g}}\nabla^C_\mu\left(\sqrt{-g}P_\alpha^{\ \beta[\mu\nu]}\right) + S^\lambda_{\ \mu\alpha}P_\lambda^{\ \beta[\mu\nu]} - S^\beta_{\ \mu\lambda}P_\alpha^{\ \lambda[\mu\nu]},$$

or equivalently, before splitting,

$$\kappa^2 H_\alpha^{\ \nu\beta} = -\frac{1}{\sqrt{-g}}\nabla_\mu\left(\sqrt{-g}P_\alpha^{\ \beta[\mu\nu]}\right) + S^\nu_{\ \sigma\rho}P_\alpha^{\ \beta\sigma\rho} + 2S^\sigma_{\ \sigma\mu}P_\alpha^{\ \beta[\mu\nu]}.$$

Trick used to simplify the torsion terms

One shifts the connection,

$$\tilde{\Gamma}^\lambda_{\ \mu\nu} = \Gamma^\lambda_{\ \mu\nu} + \alpha\,\delta^\lambda_{\ \nu}S^\sigma_{\ \sigma\mu}, \qquad \alpha = \tfrac{2}{3},$$

chosen so that the new torsion is traceless, $\tilde{S}^\sigma_{\ \sigma\nu} = 0$. In these variables the symmetric and antisymmetric parts of the connection equation decouple.

Why this matters

It shows the torsion sector is not an afterthought: it has its own structure, and the standard torsionless results are recovered as a special case.

[A]=arXiv:1212.6393, Eqs. (1.21)–(1.31).

Backup 2: the Two Branches of σ_1

$$\sigma_1 = \frac{f_R}{2} \pm \sqrt{2f_Q}\,\sqrt{\lambda^2 - \kappa^2(\rho + P)}.$$

- The branch starting at $\sigma_1 = 1$ (i.e. σ_1^+) reproduces GR at low density.
- At high density the square root vanishes; beyond that point one must continue through σ_1^- to keep σ_1 continuous and differentiable.
- This is not a new degree of freedom — just a choice of root of the same algebraic equation.

Why the branch matters for the bounce

The bounce in $f(\mathcal{R}, \mathcal{Q})$ can occur at $\sigma_1 = 0$, which lies *on the negative branch*. Without gluing the two branches one would not see the regular bouncing solutions at all.

For the radiation model the two branches meet at $\kappa^2\rho = R_P/6$, where $\sigma_1^{\pm} = \frac{1}{2}$ — this point is the "seam".

Details and the explicit $\sigma_1^{\pm}(\rho)$: [O], Secs. 1.5.6–1.5.8 and Fig. 1.3.

Backup 3: Glossary of Formalisms and Symbols

Name	What is assumed
Metric	$\Gamma = L(g)$, $T = 0$; only g dynamical
Palatini	g, Γ independent, both dynamical
Einstein–Cartan	Palatini + torsion kept
Teleparallel	curvature $= 0$; torsion carries gravity
Sym. TP	curvature $=$ torsion $= 0$; non-metricity

- ${L^\lambda}_{\mu\nu}$: Christoffel symbol of the *physical* metric g.
- ${\Gamma^\lambda}_{\mu\nu}$: independent connection.
- ${A^\lambda}_{\mu\nu} = \Gamma - L$: the extra tensor field.
- $h_{\mu\nu}$ / $q_{\mu\nu}$: auxiliary metric compatible with Γ.
- ${P_\mu}^\nu$ [M] $= {B_\mu}^\nu$ [O] $= R_{\mu\alpha}g^{\alpha\nu}$.
- $\hat{\Sigma} = f_R\hat{I} + 2f_Q\hat{P}$, and ${\Omega_\mu}^\nu = \hat{\Sigma}/\sqrt{\det\hat{\Sigma}}$.
- σ_1, σ_2: time/space dressing functions; $\Omega \equiv \sqrt{\sigma_1\sigma_2}$, $S \equiv \sigma_2^2/\Omega$.